

# Determining the smallest quantum of electric charge

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**Abstract:** Using a modern adaptation of Millikan's oil drop experiment on a smaller scale, the terminal falling and rising velocities for five individual oil droplets were determined in the presence of gravitational and electric fields. The magnitude of elementary charge was deduced from this to be  $(1.96 \pm 0.002) \times 10^{-19}$  C, with a weighted mean of  $(1.955 \pm 0.008) \times 10^{-19}$  C. The obtained value does not lie within the limits of uncertainty of the accepted value of  $(1.60 \pm 0.000049) \times 10^{-19}$  C, suggesting that there were systematic and random errors present in the procedure. These errors are suggested, as too are solutions and modifications to remove them in future experiments.

## Introduction

The aim of the experiment was to determine the smallest magnitude of the fundamental quantum of charge. The experimental method is a modern variation of that employed by Robert Millikan and Harvey Fletcher, who in 1909 showed that charged oil droplets carried indivisible multiples of a common value of charge, the charge carried by an electron [1].

In the experiment, oil droplets with an average radius of  $(5.03 \pm 0.02) \mu\text{m}$ , were allowed to fall freely in the Earth's gravitational field through air between two aluminium plates of a capacitor, eventually reaching their terminal velocity ( $u$ ). As the velocity of the oil droplets increased so too did the viscous drag force experienced until the forces due to Archimedean upthrust,  $F_u$ , and viscous drag,  $F_d$ , were in equilibrium with the force due to gravity,  $F_g$ . The equation of motion when the net force on the falling oil droplet is zero is described by equation (1.1):

$$6\pi\eta au = \frac{4}{3}\pi a^3(\rho - \sigma)g \quad (1.1)$$

Where  $\eta$  = the coefficient of viscosity,  $a$  = the radius of the oil droplet,  $\rho$  = the density of the oil droplet,  $\sigma$  = the density of air and  $g$ =the acceleration due to gravity.

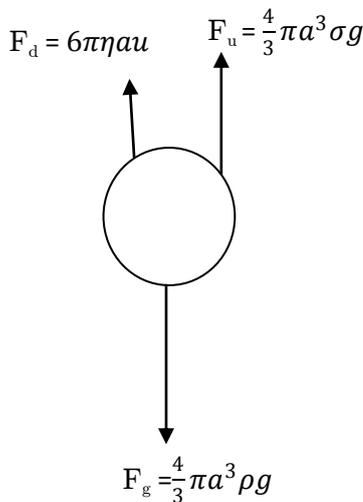


Figure 1: Forces acting on the oil droplet with no application of electric field.

Knowing the distance through which the oil droplet fell and the time taken for descent enabled the calculation of terminal velocity ( $u$ ) and hence radius of the oil droplet:

$$a = \left[ \frac{9\eta u}{2(\rho - \sigma)g} \right]^{\frac{1}{2}} \quad (1.2)$$

The application of  $(0.6 \pm 0.0005)$  kV across the plates of the capacitor gave rise to an electric field with polarity such that the electrostatic force on the oil droplet opposed the force due to gravity. Under the influence of this electric field, the oil droplet experienced net movement upwards, reaching its terminal rising velocity ( $v$ ) once the downward gravitational and viscous drag forces were in equilibrium with the Archimedean upthrust and Coulombic forces, as is shown by equation (1.3):

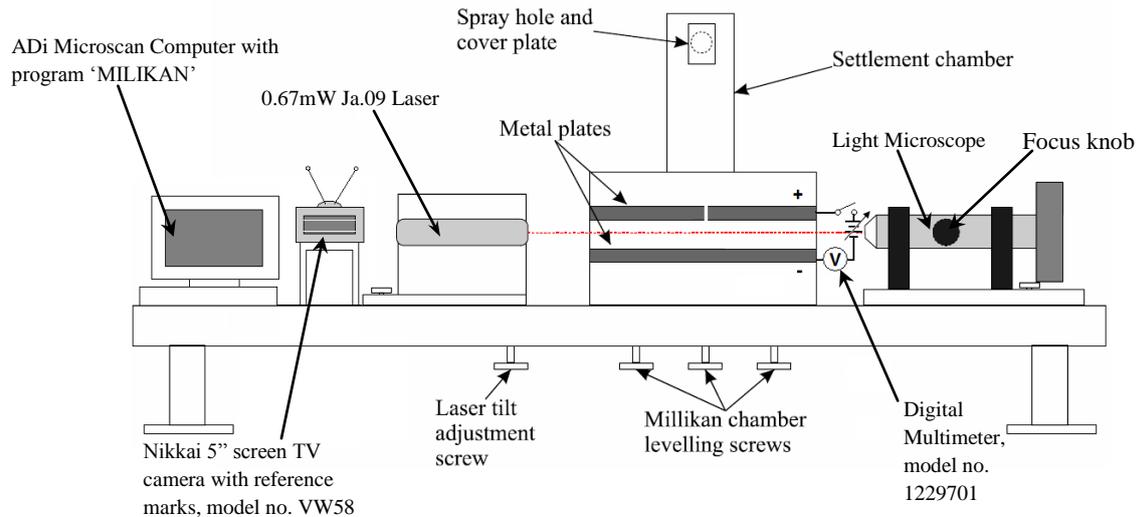
$$6\pi\eta av + \frac{4}{3}\pi a^3\rho g = q\frac{V}{d} + \frac{4}{3}\pi a^3\sigma g \quad (1.3)$$

Where  $d$  is the separation of the capacitor plates and  $V$  is the voltage applied across them. However, since the oil droplets were not travelling through a continuous medium, but one composed of discrete air molecules, the difference in size between the air molecules and the oil droplets required a modification of Stoke's law. Taking consideration for the fact that inhomogeneties in the medium will be somewhat comparable with the size of the oil droplets, equation (1.3) is rearranged and modified to:

$$q_c = \frac{q}{\left(1 + \frac{b}{pa}\right)^{3/2}} \quad (1.4)$$

Where  $p$  = atmospheric pressure and  $b$ = the correction constant. The viscosity of dry air is proportional to temperature and therefore pressure [3]. Atmospheric pressure was noted to be: 757.7 mmHg.

## Method



**Figure 2: Schematic diagram of the Millikan Chamber and associated apparatus [2]**

Instead of using the hazardous X-ray source in Millikan's original experiment, oil droplets were sprayed into the settlement chamber from the atomiser, acquiring excess electrons via friction with the nozzle. With the cover plate closed to prevent any loss of oil droplets, the oil droplets then fell through the small hole in the settlement chamber, and again into the space between the two plates of the capacitor. To view these oil droplets, all electrical connections were checked to be secure with the power off, and the TV camera and ADi Microscan Computer were switched on, giving a magnified image by a factor of  $\approx 1000$ . The force exerted by an electric field on a particle carrying one or several excess electrons is given by the following relationship:

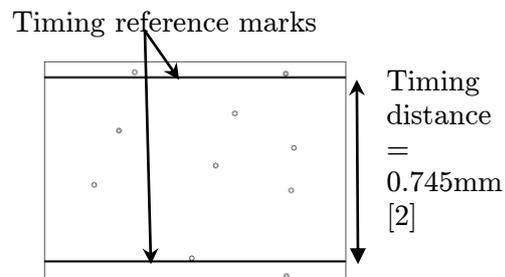
$$F = \frac{Vq}{d} \quad (1.5)$$

A high potential difference and small separation distance is therefore needed to produce an upward electrostatic force that produces net upward motion. The electrostatic force was created by a high voltage (HV),  $(0.6 \pm 0.005)$  kV, applied across the metal plates, separated by a distance  $d = 4.76$  mm [2]. Note that the HV had been attenuated by a factor of 1000 before being measured with a digital multimeter (DVM), limiting any risk of exposure.

Using the hand-held HV switch, the oil droplets were observed to fall downwards with the HV supply off, and upwards with the HV supply on, in the vertical plane of

the TV monitor. To reduce the number of oil droplets visible on the screen, the HV supply was switched on and off in 10 second intervals so that heavier, neutral oil droplets settled at the bottom of the Millikan chamber. As equation (1.4) suggests, oil droplets with slower rising and falling terminal velocities will carry smaller integer multiples of elementary charge, therefore slower droplets, located at the centre of the TV monitor were selected for observation.

With program 'MILIKAN' initiated, the selected oil droplet was then raised above the first reference mark via the application of  $(0.6 \pm 0.0005)$  kV, the uncertainty arising from the precision of the DVM. As the centre of the oil droplet was parallel to the first reference mark, 'ENTER' was pressed on the keyboard, and pressed again once the oil droplet had passed through the second reference mark.



**Figure 3: Oil droplets as observed on TV monitor.**

The same process was then repeated but with the application of the HV across the metal plates causing upward motion of the oil droplet through the reference marks. The computer then generated rise times and fall times via its internal stopwatch from which the corrected charge on the oil droplet could be inferred. It was noted that the laser light diffracted off of the oil

droplets in several experiments, giving the oil drops a blurred appearance making them difficult to track. This was rectified by adjusting the focus knob on the microscope for better contrast. Unfortunately several observations had to be rejected due to oil droplets disappearing from view.

## Results & Analysis

**Table 1: Summarised results for five different oil droplets falling and rising through a gravitational and electric field.**

Fall time /s	Rise time /s	Radius of droplet /x10 <sup>-7</sup> m	Corrected charge / x10 <sup>-19</sup> C	Standard deviation, σ /x10 <sup>-21</sup>
28.62	6.32	4.98	1.94	3.12
28.12	6.26	5.02	1.95	
28.23	6.31	5.01	1.95	
26.48	6.54	5.18	2.01	
28.51	6.64	4.99	1.94	
25.98	6.75	5.23	2.03	5.35
26.53	6.64	5.17	2.01	
26.14	6.65	5.21	2.02	
24.39	6.48	5.39	2.10	
28.29	6.81	5.01	1.95	
26.91	6.92	5.13	2.00	4.36
27.03	6.48	5.12	1.99	
29.05	6.32	4.94	1.92	
27.96	6.15	5.04	1.96	
29.82	6.26	4.88	1.90	
25.65	6.21	5.26	2.04	8.13
30.15	6.26	4.85	1.89	
29.60	6.10	4.90	1.90	
32.30	6.42	4.69	1.82	
29.60	6.04	4.90	1.90	
27.90	6.31	5.04	1.96	3.45
28.84	6.32	4.96	1.93	
28.73	6.04	4.97	1.93	
27.68	6.32	5.06	1.97	
30.32	5.99	4.84	1.88	

The corrected charge for each oil droplet was determined using equation (1.4), and its associated uncertainty was deduced using Microsoft Excel's average and standard deviation functions. Assuming the constants to possess negligible uncertainties, the error due to human reaction time when pressing 'ENTER' dominates in this procedure and can be propagated using the standard error on the mean:

$$\Delta \bar{q}_c = \frac{\sigma_{qc}}{\sqrt{N}} \quad (1.6)$$

Where  $\sigma_{qc}$  is the standard deviation of the corrected charge and N is the number of pairs of timings. Given that there were five pairs of timings for each oil droplet, and that the standard deviation of each oil droplet had been calculated, the mean corrected charge was found to be  $(1.96 \pm 0.02) \times 10^{-19}$  C. Dividing each of the values for corrected charge by the accepted value of the charge on an electron:  $(1.60 \pm 0.0000049) \times 10^{-19}$  C [3], gave a value close to 1 each time, showing that the smallest quantum of electric charge had been determined.

To improve the accuracy of the mean corrected charge so that values with a smaller uncertainty contributed more to the final average, the weighted mean was then calculated for each of the droplets.

$$\text{Given by: } \bar{e} = \frac{\sum_{i=1}^n w_i q_i}{\sum_{i=1}^n w_i} \quad (1.7)$$

The associated uncertainty in the weighted mean was then determined using:

$$\sigma_e = \sqrt{\frac{1}{\sum \left( \frac{1}{\Delta q_c^2} \right)}} \quad (1.8)$$

The final weighted mean of the smallest quantum of electric charge was found to be:

$$(1.955 \pm 0.008) \times 10^{-19} \text{ C}$$

This compares with the tabulated value of:

$$(1.60 \pm 0.0000049) \times 10^{-19} \text{ C [3]}$$

Although the experimentally obtained value is of the same order of magnitude, it does not lie within the limits of uncertainty of the accepted value of 'e'.

The measured value of 'e' differs from the accepted value by 35.3 times the experimental error, which is suggestive of large systematic error in the experiment.

Possible sources of systematic error could include:

- Insufficient time allocated for the oil to achieve terminal velocity and so the equations (1.0 – 1.4) did not apply for the complete duration of the descent/ascent.
- The effects due to Brownian motion were significant enough to disrupt the time that would be taken to fall/rise through the timing distance, imparting additional forces on the oil droplet which are not accounted for by Stoke's law, Coulomb's law, gravity or upthrust. This effect could be eradicated by performing the experiment in a vacuum, where equation (1.4) would become  $q_c = \frac{4\pi r^3 \rho g}{3V}$ , a true vacuum however is hard to achieve.
- The oil droplets charge may have changed at some stage in its motion. The laser's power was only 0.67milliwatts and should not have had enough energy to ionise any of the oil droplets. This uncertainty can be eliminated by purposely changing the charge on the oil droplet during its motion. If an ionization source such as thorium-232 is placed near the drop, the oil droplet's charge can be changed and the new fall/rise time measured [3]. The resulting corrected charge value however should still be a multiple of some smallest charge  $e$ , close to the value that has already been calculated.
- Distortions in the electric field – The electric field produced may not have been completely uniform due to the plates not being completely parallel or the aperture in the upper plate creating a small disturbance in the electric field. Future experiments could involve placing the plates of the capacitor vertically, rather than horizontally.

The human perception error dominates in the procedure and so in the future, the

human element of observation would need to be removed.

A modification to the procedure could involve placing light gates at each of the reference marks. As the oil droplet passes through each of the light gates, the signal between transmitter and receiver would be momentarily interrupted and the time between the interruption of the first and second light gate could then be determined. A similar approach has been devised by scientists at the Stanford Linear Accelerator Centre (SLAC), where droplets of equal radius are created using micro-machined ejectors and their motion observed using an automated imaging system that uses CCD cameras connected to powerful computers [4]. The technology for this however was not available.

### Conclusion

In summary, the weighted mean of the charge of an electron was determined to be:  $(1.96 \pm 0.01) \times 10^{-19} \text{ C}$ , which differs from the accepted value of  $1.60 \times 10^{-19} \text{ C}$  by  $\approx 0.353 \times 10^{-19} \text{ C}$  or  $\approx 35.3\%$ , suggesting large sources of systematic error were present in the procedure. Although measures were taken to reduce and account for these systematic errors, it is thought that the accuracy of the procedure was limited by human perception, resolution of the microscope and the effects of Brownian motion.

### References

- [1] R. A., Millikan, *A new modification of the cloud method of determining the elementary electrical charge and the most probable value of that charge*, Phil. Mag. XIX, 6, pp. 209-228 (1910)
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- [4] M. Perl, *Fraction man*, New Scientist, 2400, pp.44 – 45 (2004)